Optimal Control and Learning in Humanoid Robotics

Justin Carpentier

Post-doc @ LAAS-CNRS

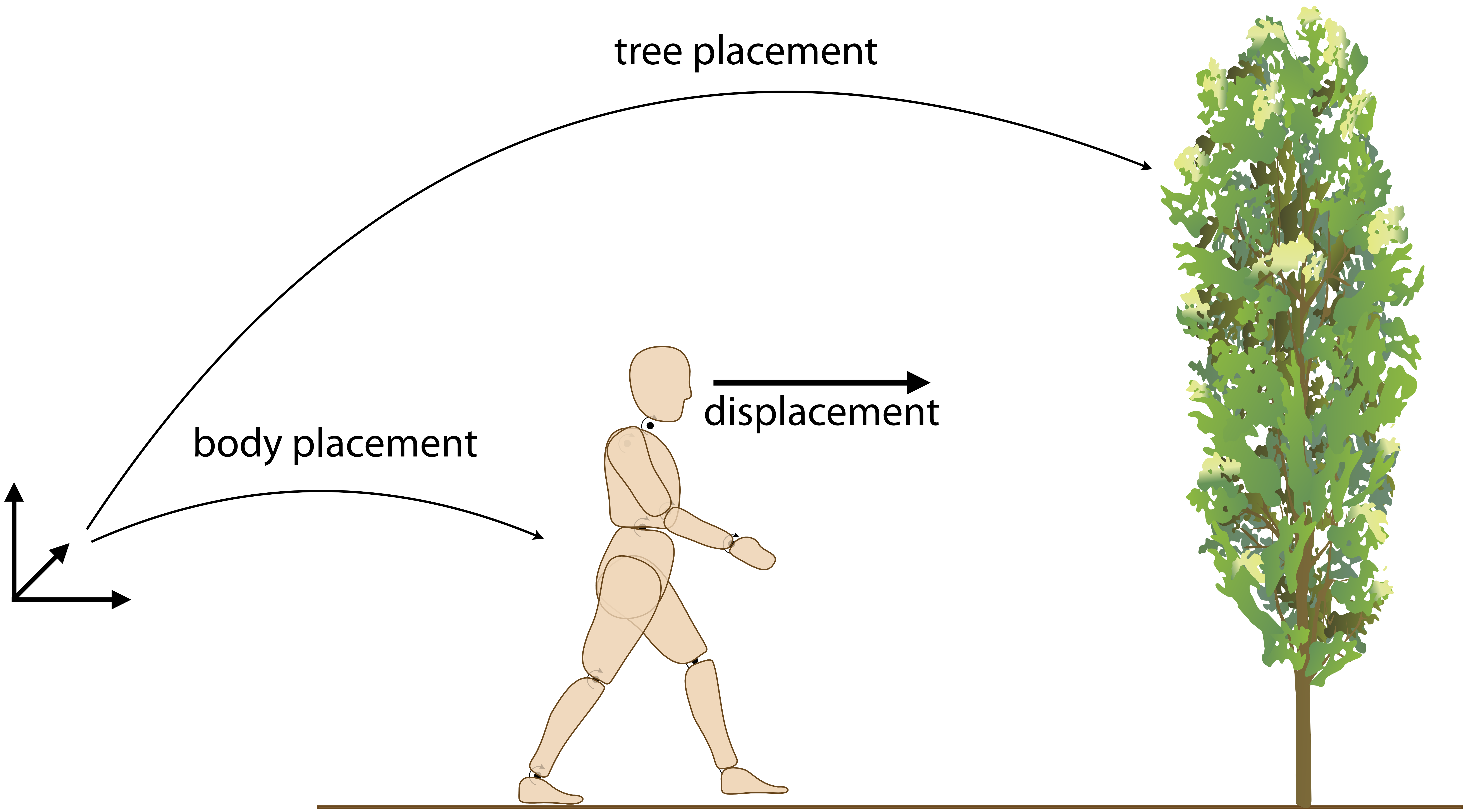
Gepetto

cnrs

Fundamentals of humanoid locomotion

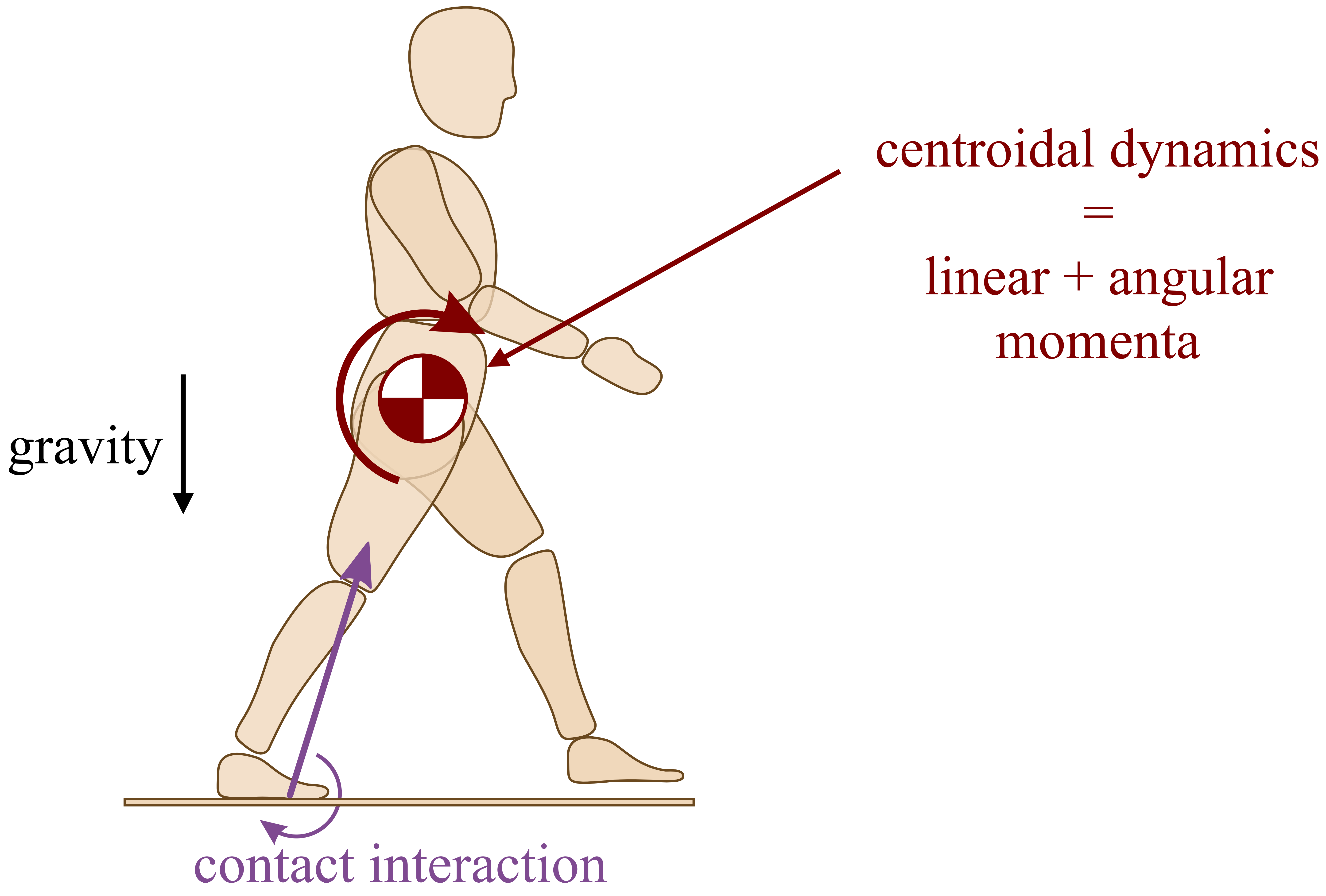
Locomotion — a geometric view

Posture x Placement



Locomotion — a dynamic view

Actuated Dynamics vs Centroidal Dynamics



The three spaces of movement

Physical space

posture x placement

Robotics

Motor space

internal actuation / contact forces

Sensorial space

proprioception / exteroception

Why the locomotion problem is difficult?

- Nonlinear and hybrid dynamics
- Under-actuated dynamics
- Unstable dynamics
- Nonholonomic and unilateral constraints
- Dimensionally of the problem
- The system must not fall

Locomotion as a Mixed-Integer Optimal Control Problem

control trajectory

state trajectory

phase duration

set of active contacts

$$\min_{\underline{\boldsymbol{x}},\underline{\boldsymbol{u}},(\Delta t_s),\Phi_s} \quad \sum_{s=1}^{S} \int_{t_s}^{t_s+\Delta t_s} \ell_{\Phi_s}(\boldsymbol{x}, \dot{\boldsymbol{x}})\, dt$$

$$\text{s.t. } \dot{\boldsymbol{x}} = f_{\Phi_s}(\boldsymbol{x}, \boldsymbol{u}), \ \forall t \in [t_s, t_s + \Delta t_s)$$

cost function

whole-body dynamics

state and control constraints

initial state is given

final state is viable

Which types of locomotion current humanoids can do?

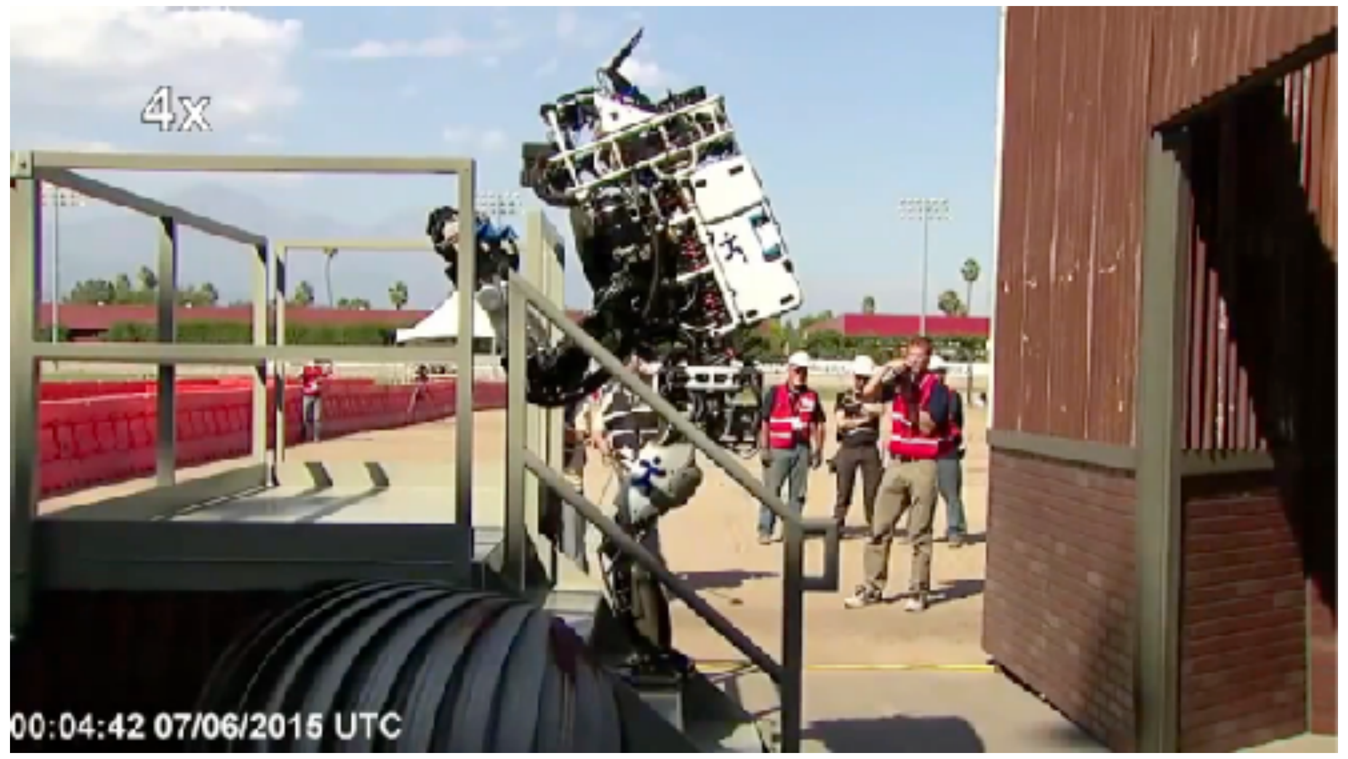
Walking

[Atlas, Boston Dynamics, '16]

Running

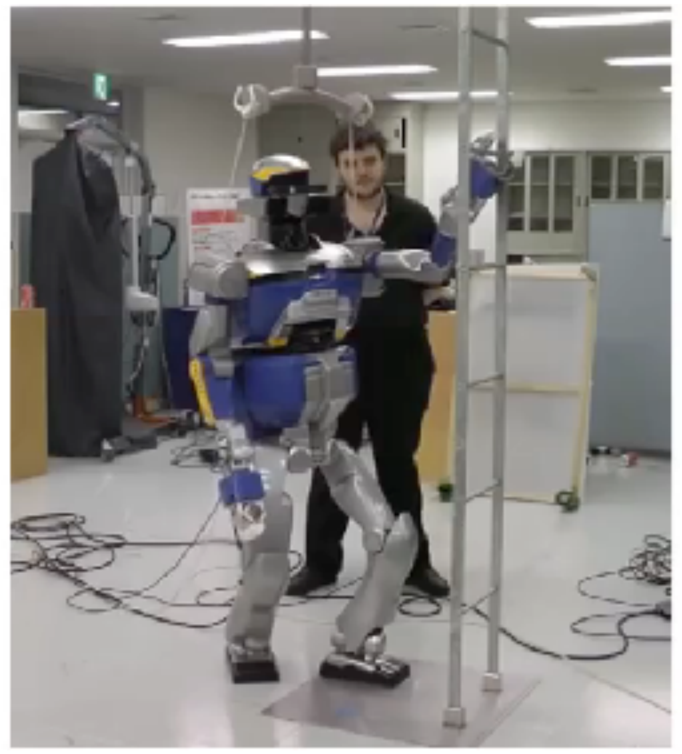
[Asimo, HONDA, '14]

Climbing stairs



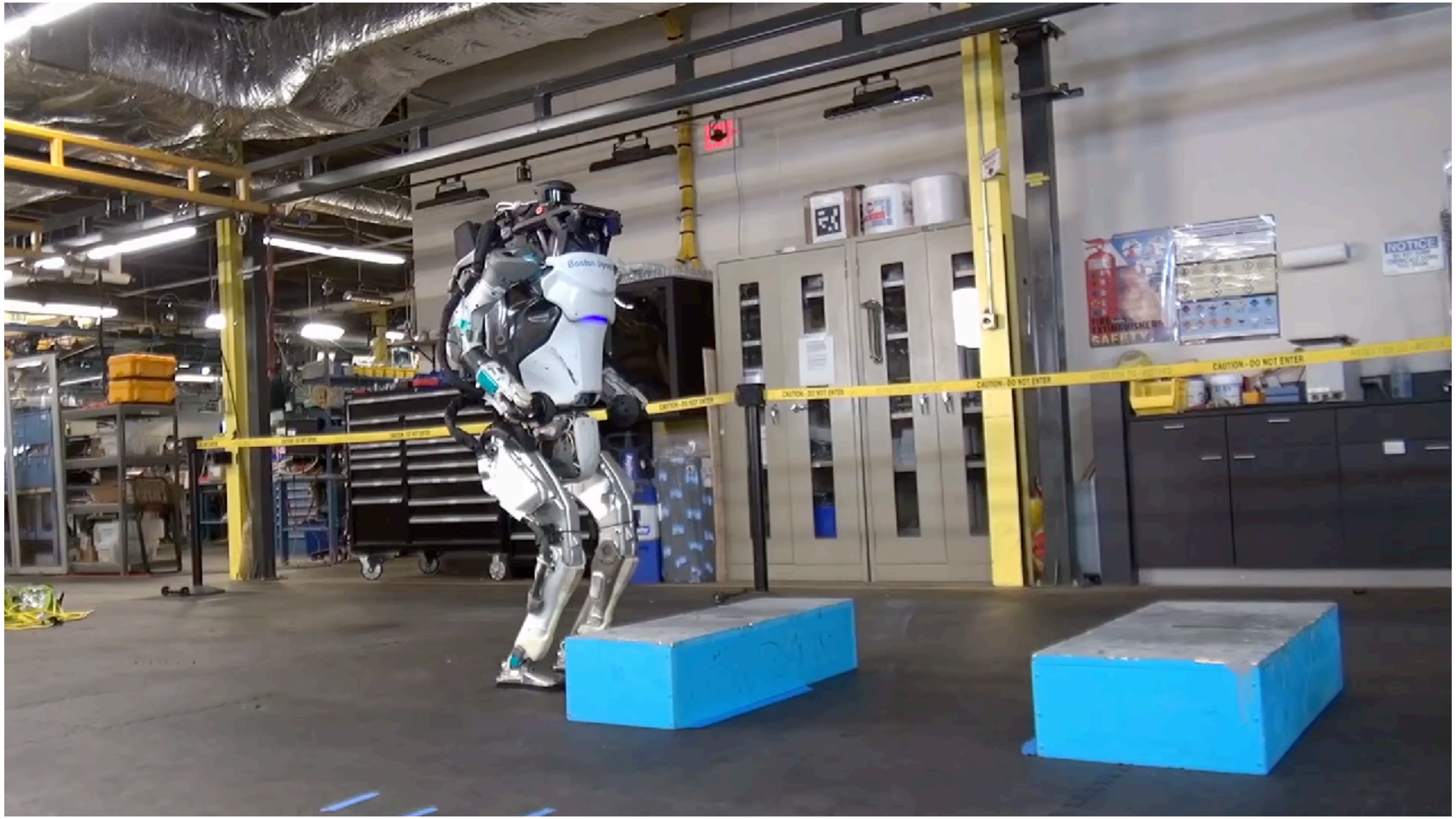
[Atlas, IHMC, '15]

Climbing ladders



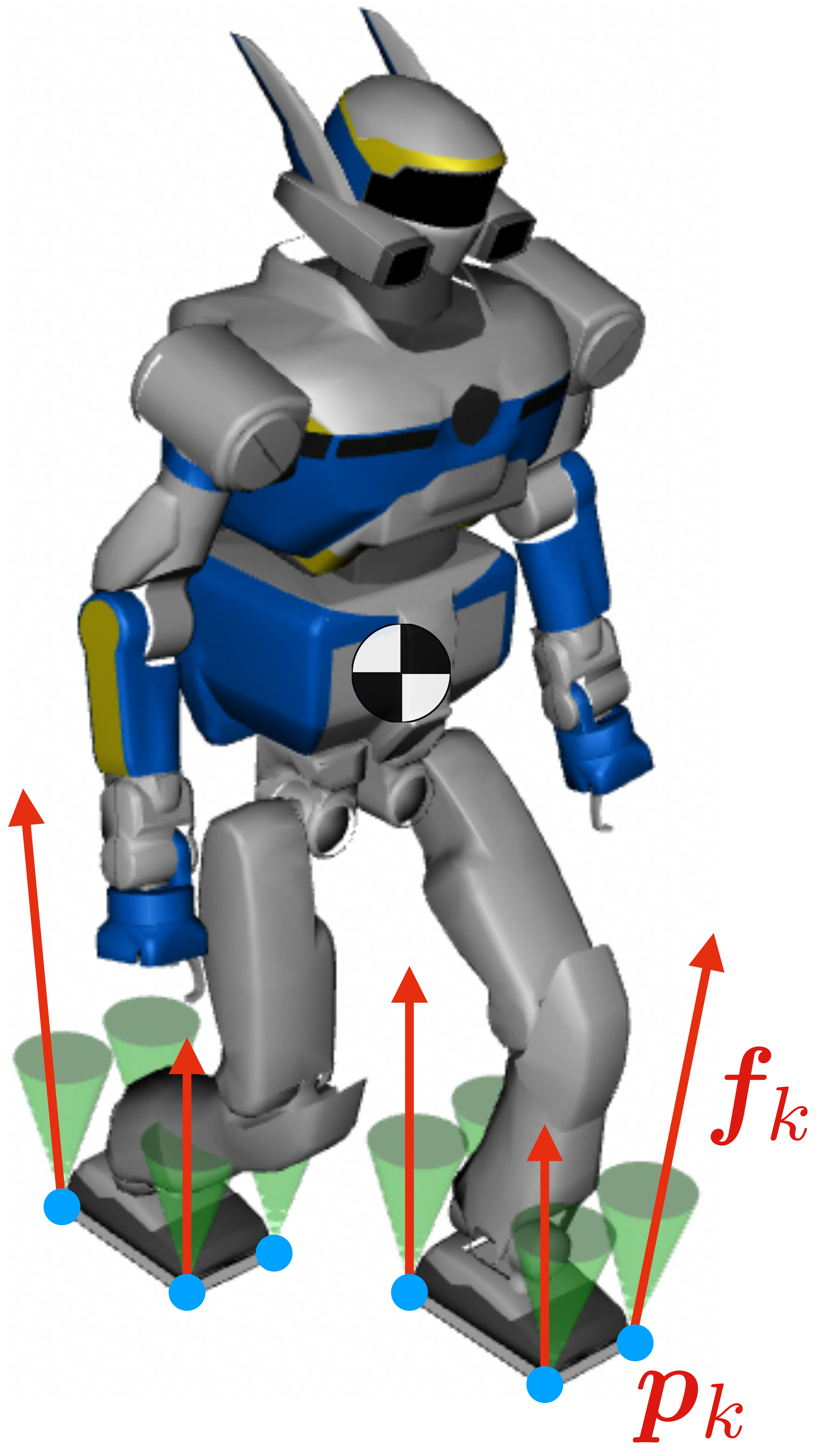
[HRP-2, JRL, '16]

And of course …



How decoupling can help?

Multi-body dynamic equations



Lagrangian dynamics of a multi-body system

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + g(\boldsymbol{q}) \;= S\boldsymbol{\tau}$$

Under-actuated dynamics

$$\begin{bmatrix} M_u \\ M_a \end{bmatrix} \ddot{\boldsymbol{q}} + \begin{bmatrix} C_u \\ C_a \end{bmatrix} \dot{\boldsymbol{q}} + \begin{bmatrix} g_u \\ g_a \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\tau} \end{bmatrix} + \begin{bmatrix} J_u^\top \\ J_a^\top \end{bmatrix} \boldsymbol{\lambda}$$

Manipulator dynamics

Two approaches to solve multi-contact locomotion problems

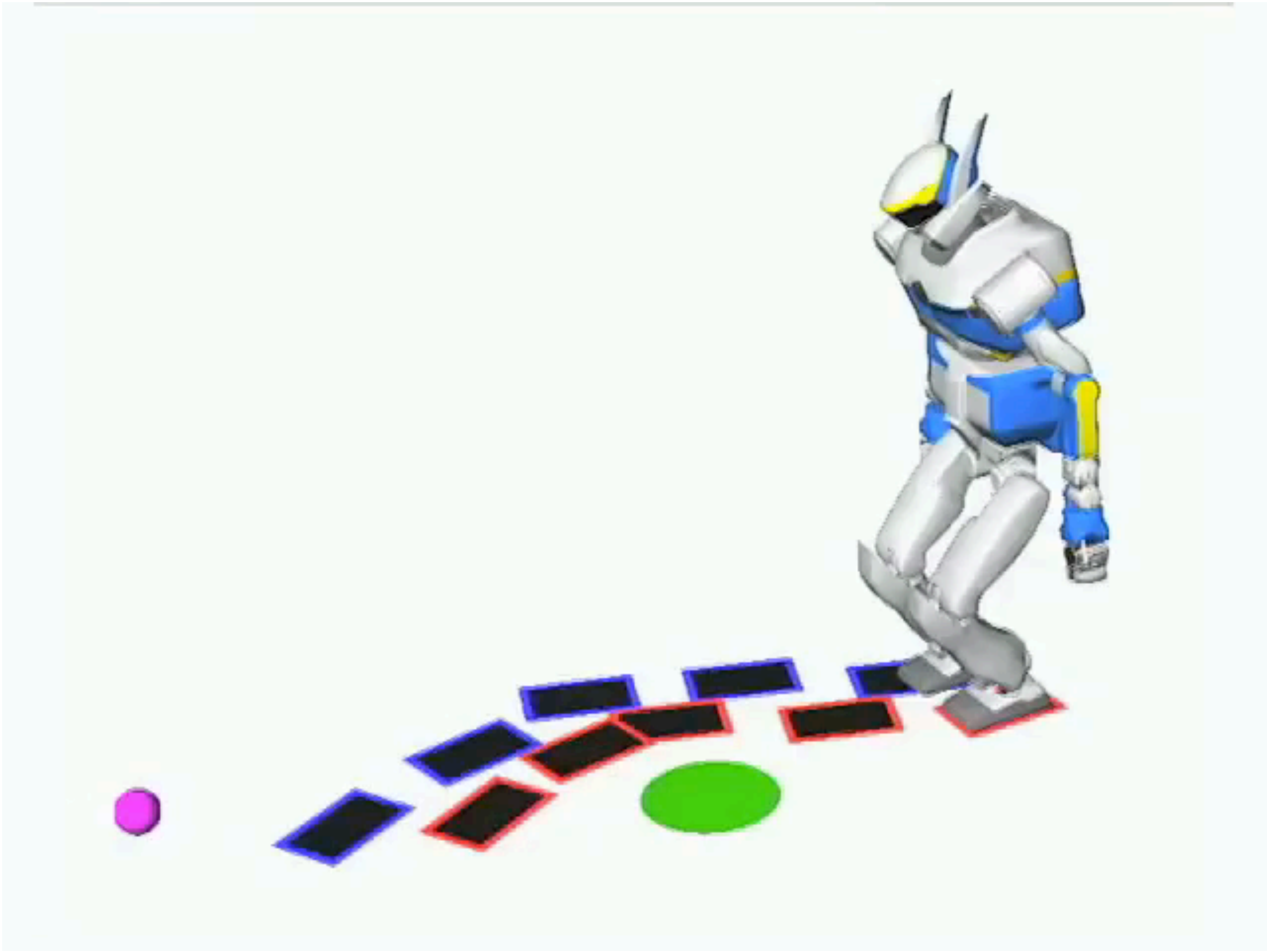
Coupled approach

Simultaneous contact planning and whole-body generation

[Erez et al., Humanoids, '13]

Decoupled approach

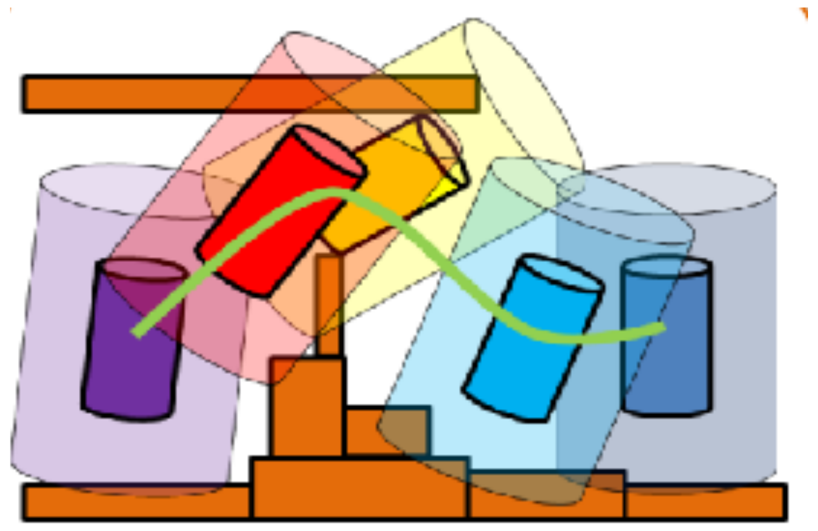
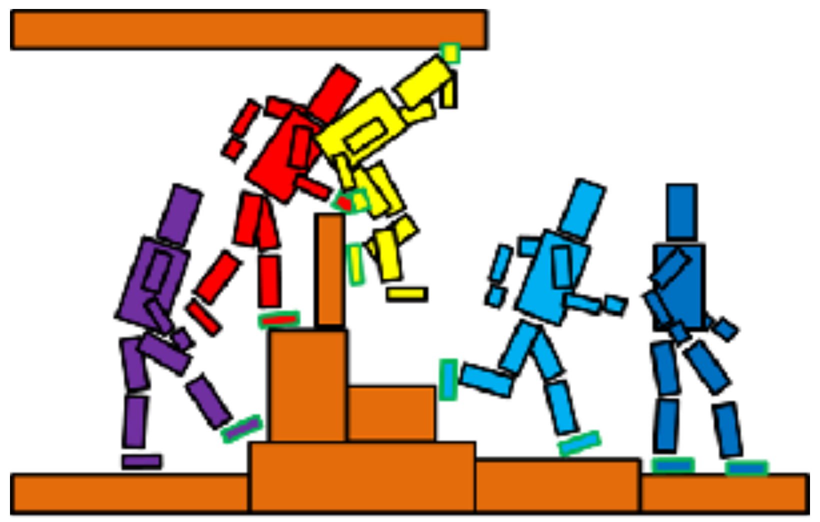
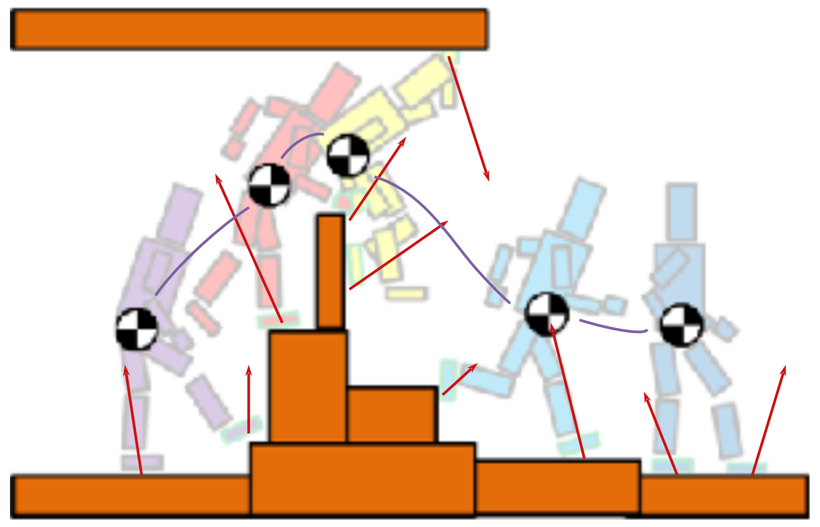
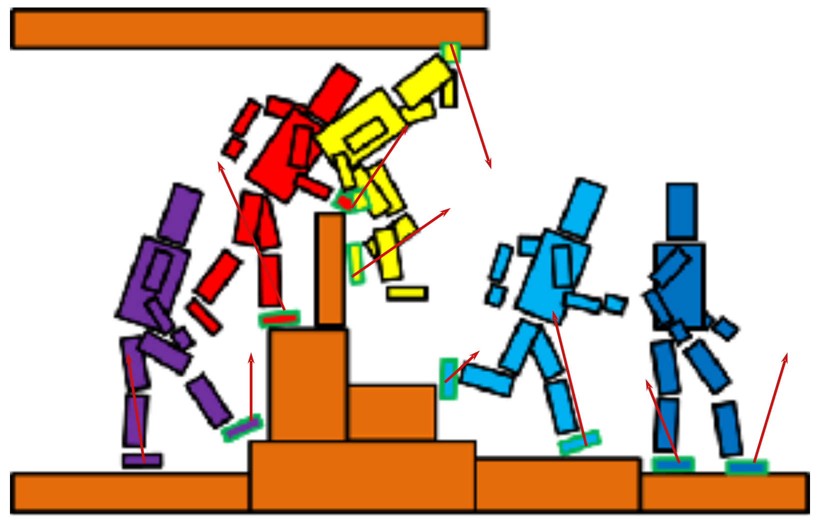
First plan contacts, second animate the contact sequence

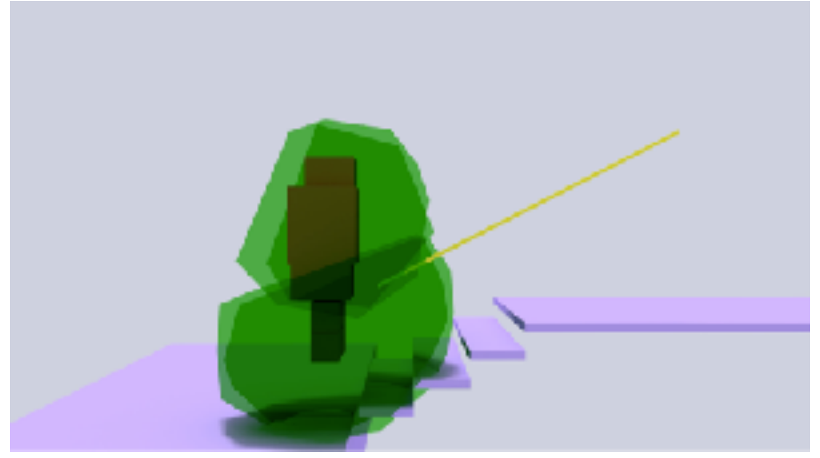
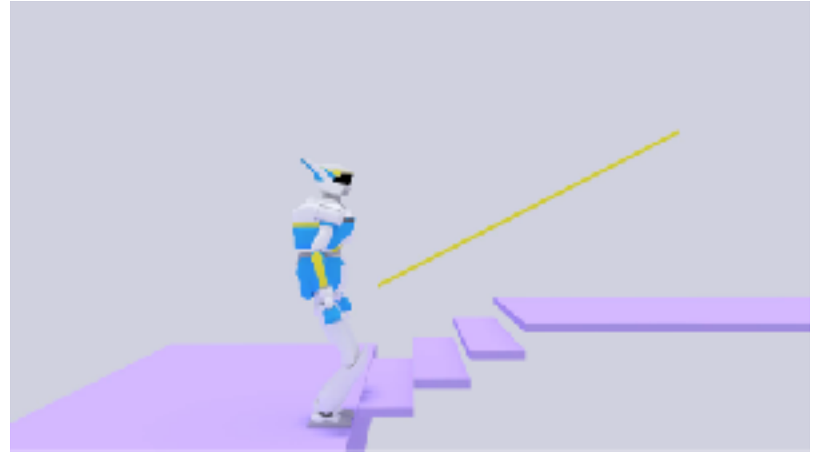
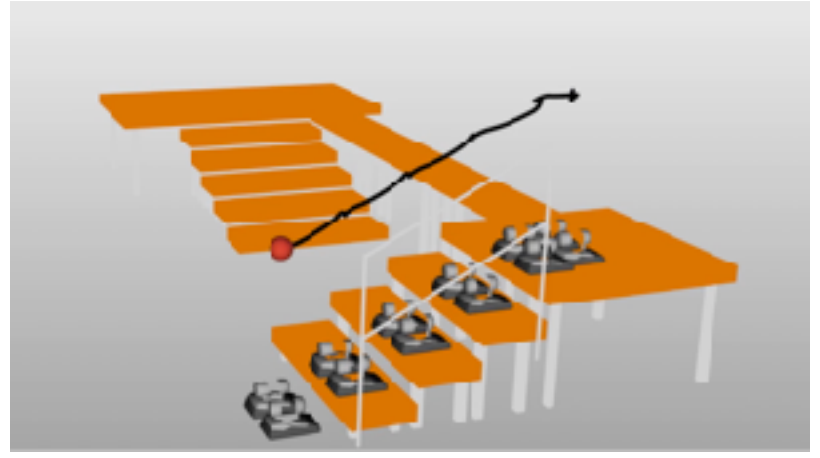


[Kanoun et al., IJRR, '11]

The multi-contact workflow

> 1 Hz	2-5 Hz	100 Hz	1-2 kHz
Reachability planner	Contact sequence planner	Centroidal motion generation	Whole-body motion generation

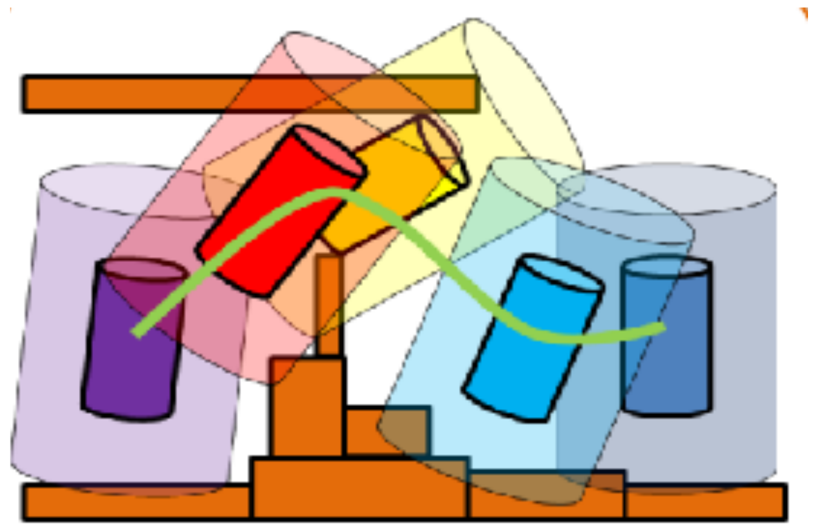
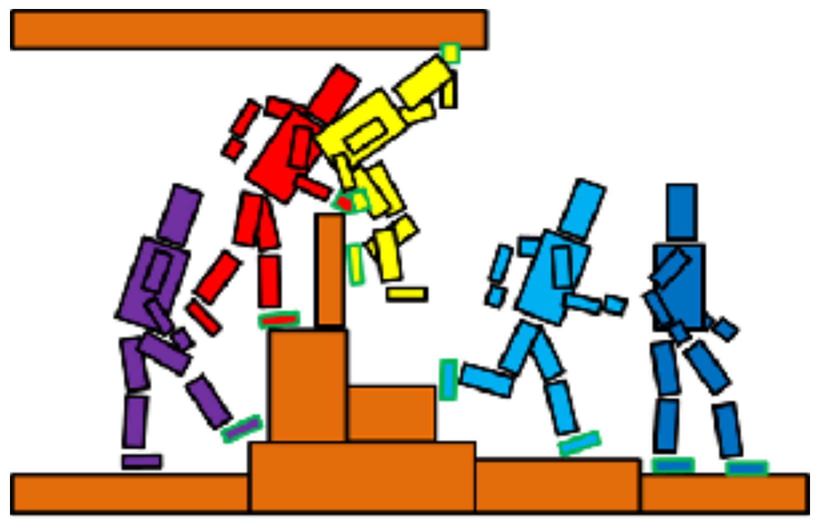
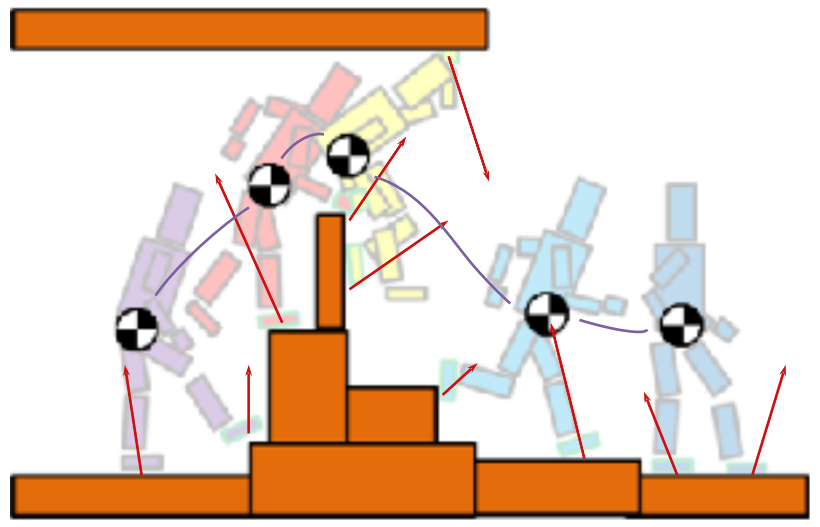
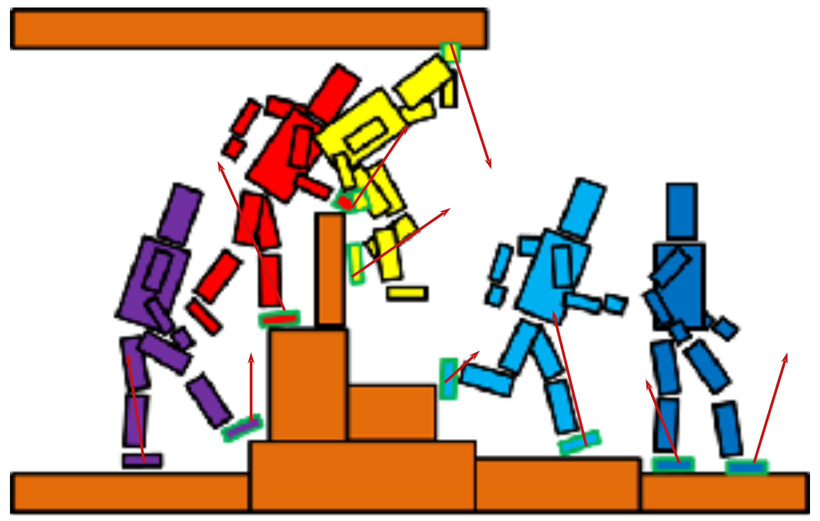





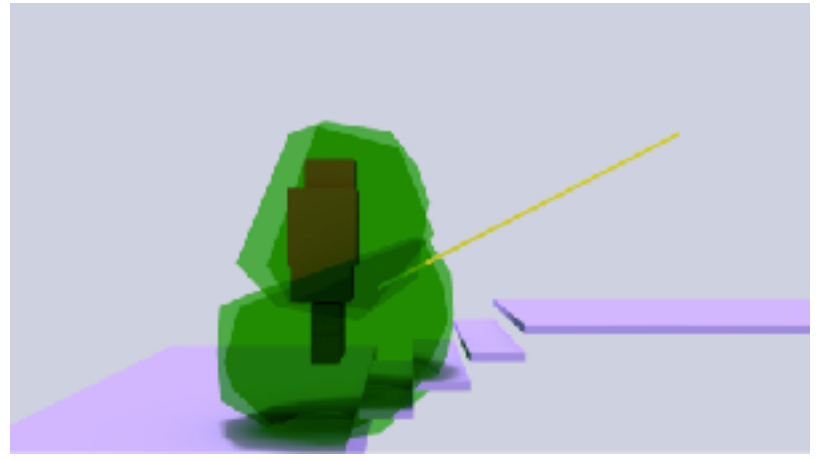
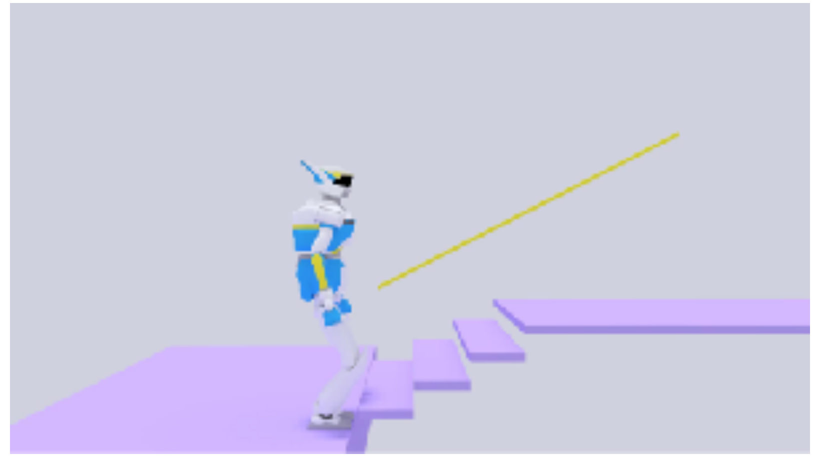
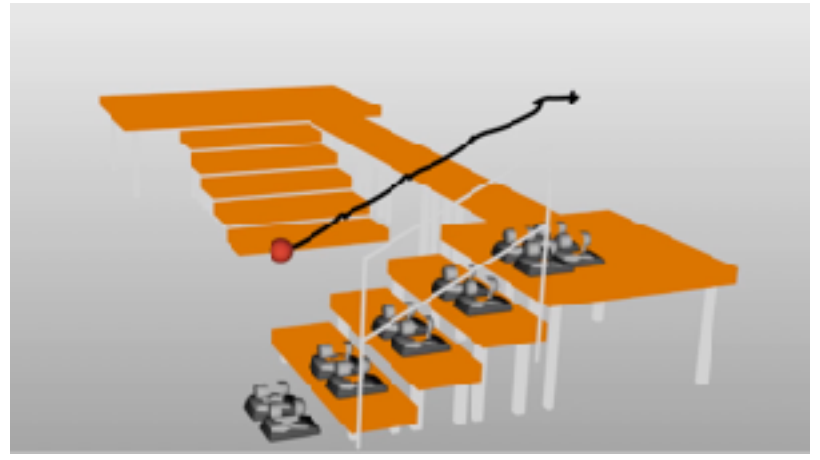





Break the complexity of the locomotion problem
by relying on **reduced/conservative** models

Contact sequence generation

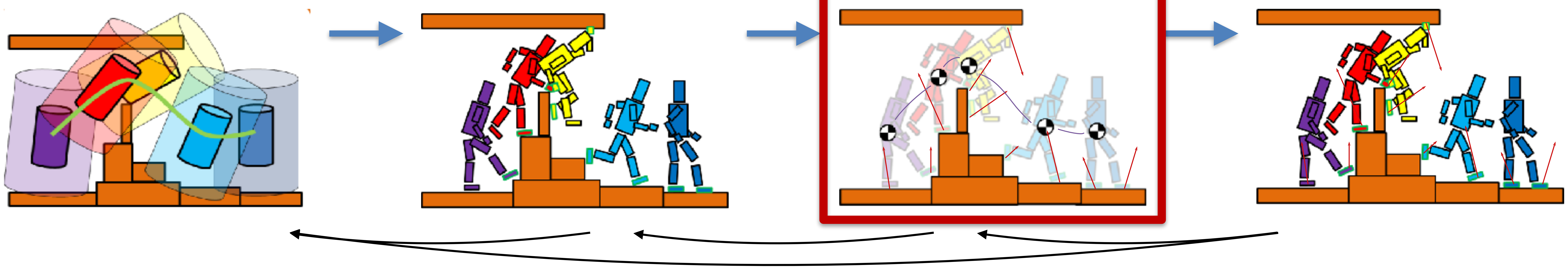
> 1 Hz	2-5 Hz	100 Hz	1-2 kHz
Root path planner	Contact sequence planner	Centroidal motion generation	Whole-body motion generation

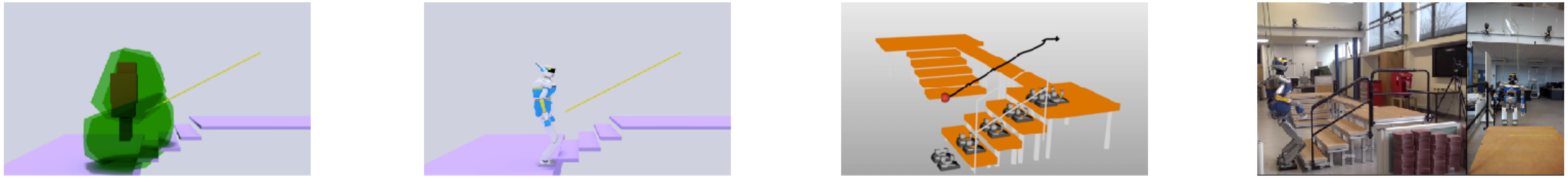





Centroidal motion generation

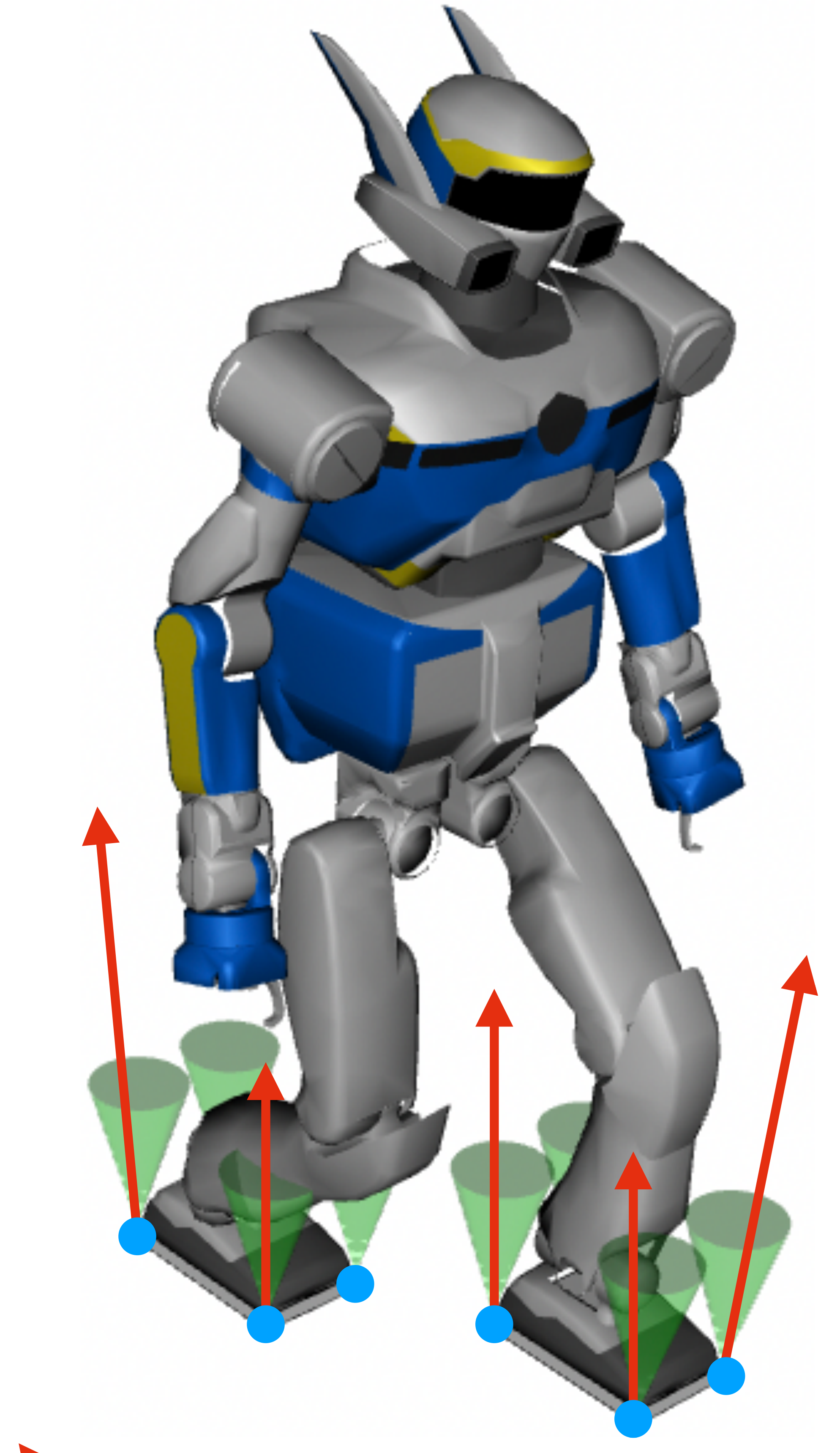
> 1 Hz	2-5 Hz	100 Hz	1-2 kHz
Root path planner	Contact sequence planner	Centroidal motion generation	Whole-body motion generation



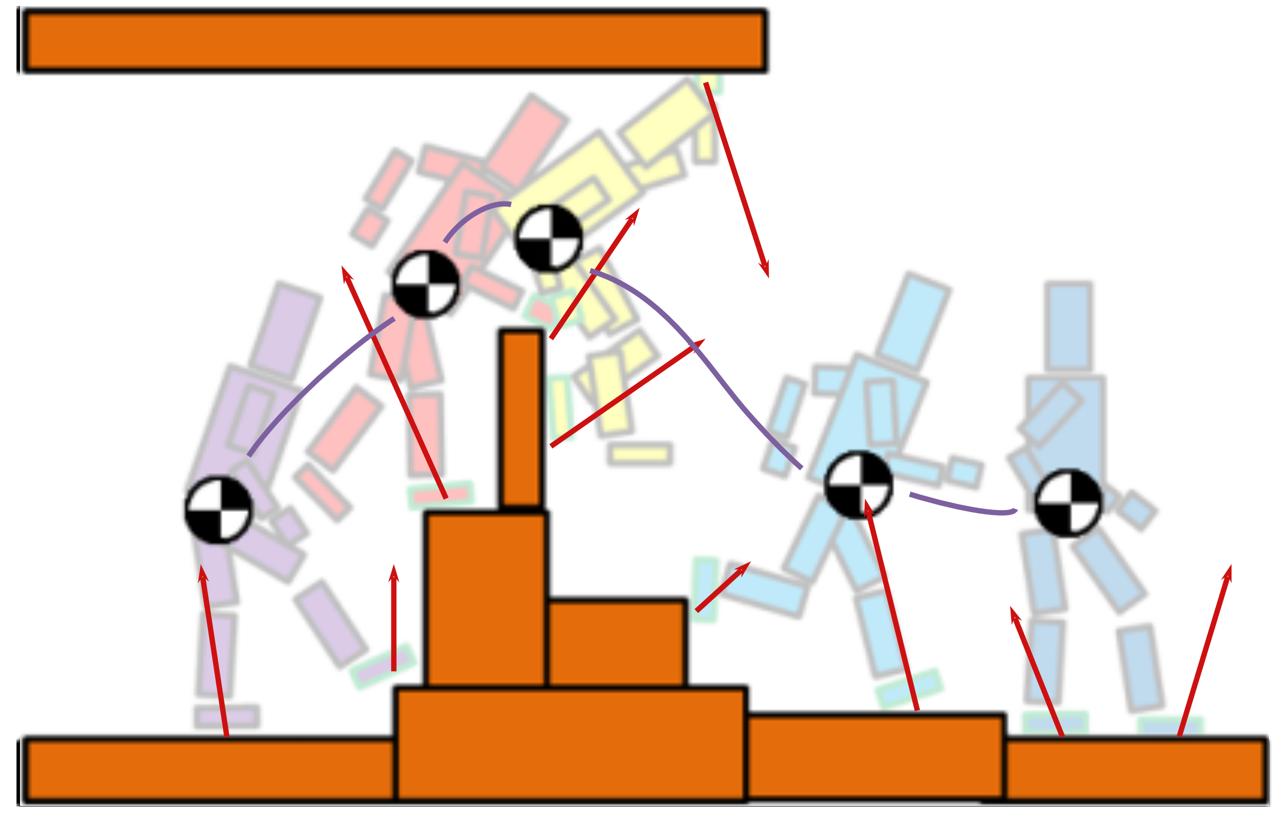
necessity of proxy constraints



We only consider the centroidal dynamics of the robot



- contact forces $\mathbf{f_k}$
- contact positions $\mathbf{p_k}$
- friction cones
- center of mass



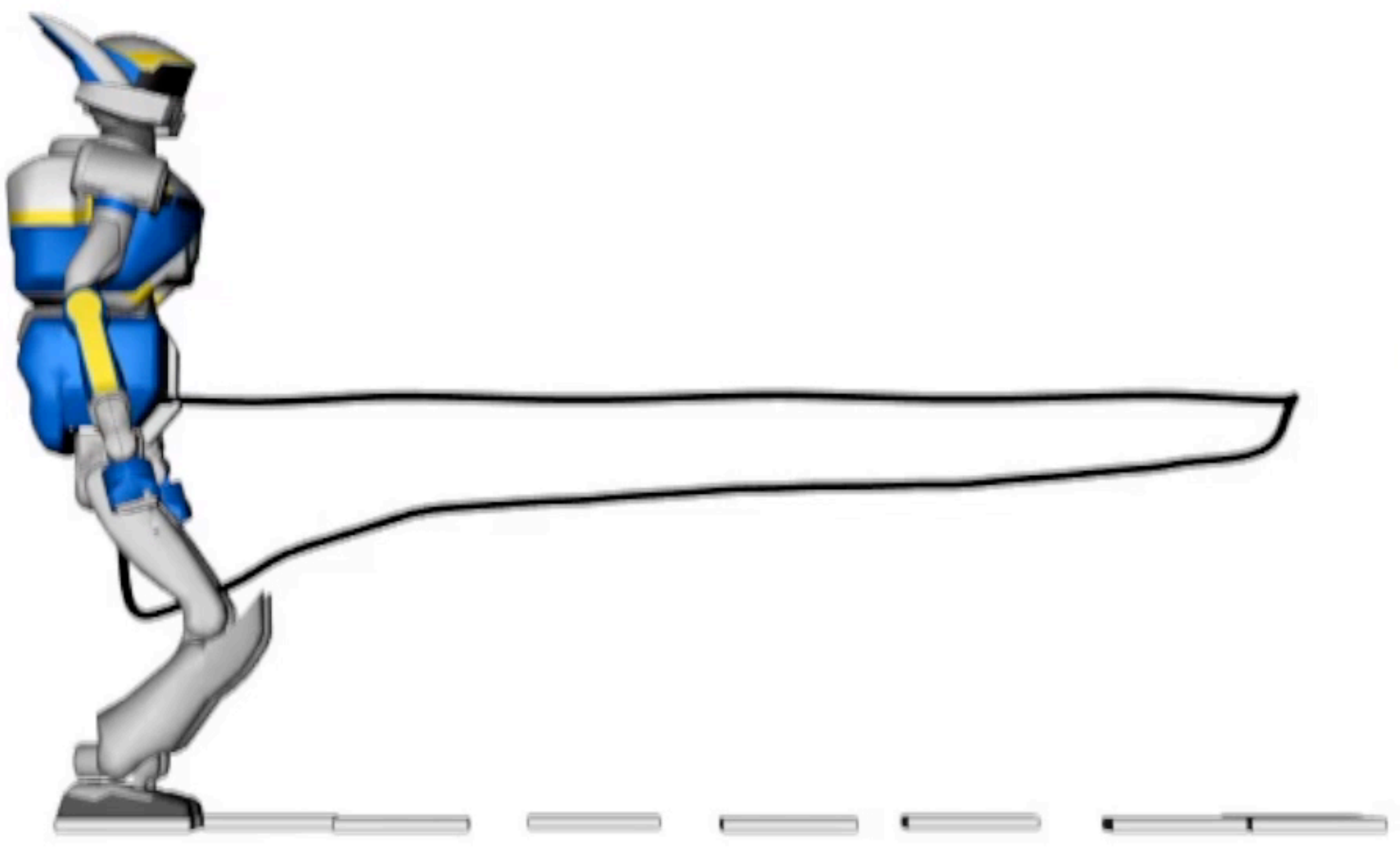
Centroidal dynamics equation

Newton and Euler equations of motion

$$\dot{\boldsymbol{h}} = \sum_{k=1}^{K} \boldsymbol{f}_k + m\boldsymbol{g}$$

$$\dot{\boldsymbol{L}}_{\boldsymbol{c}} = \sum_{k=1}^{K} (\boldsymbol{p}_k - \boldsymbol{c}) \times \boldsymbol{f}_k$$

Reduced optimal control formulation

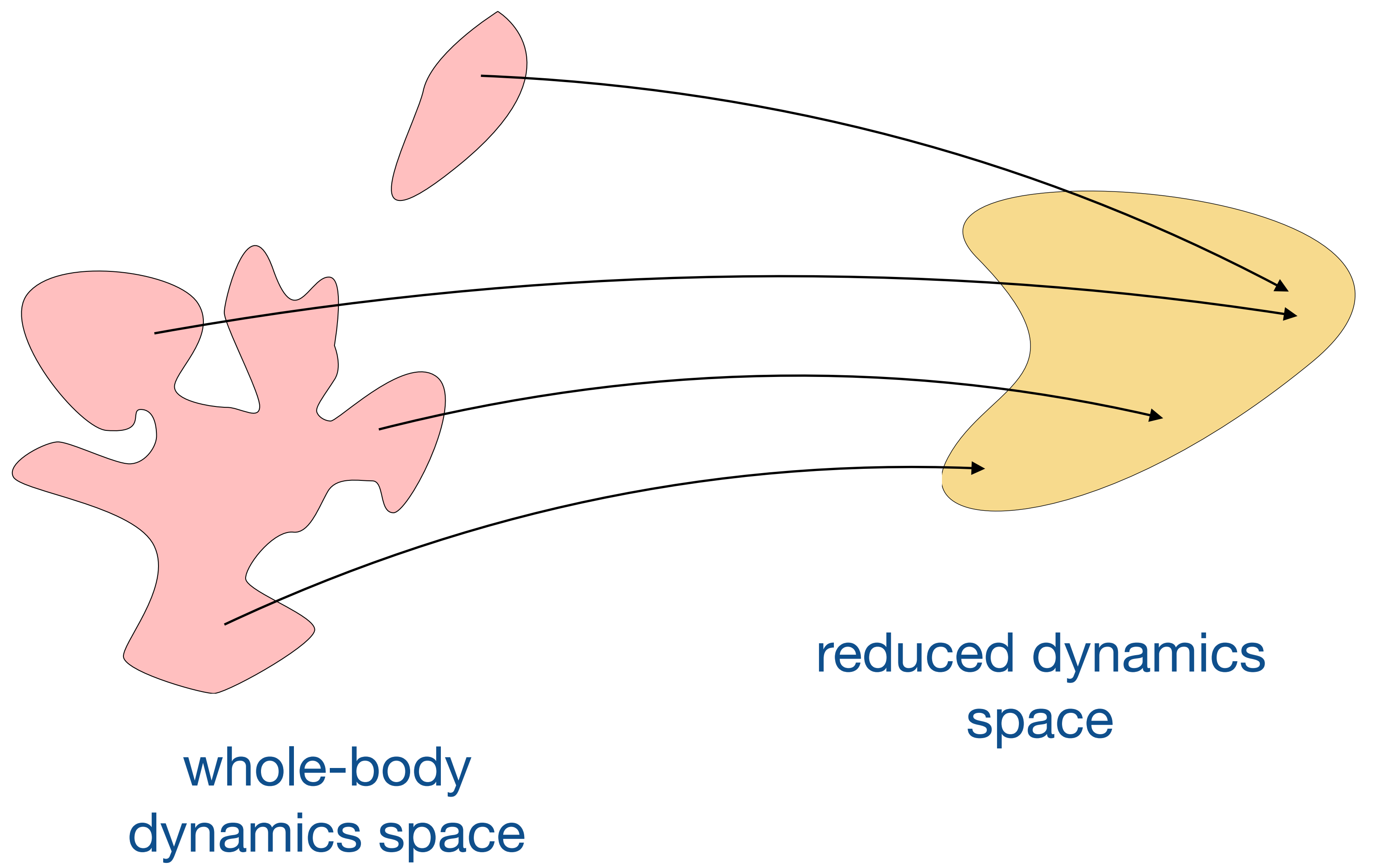


Reduced optimal control formulation with feasibility constraint

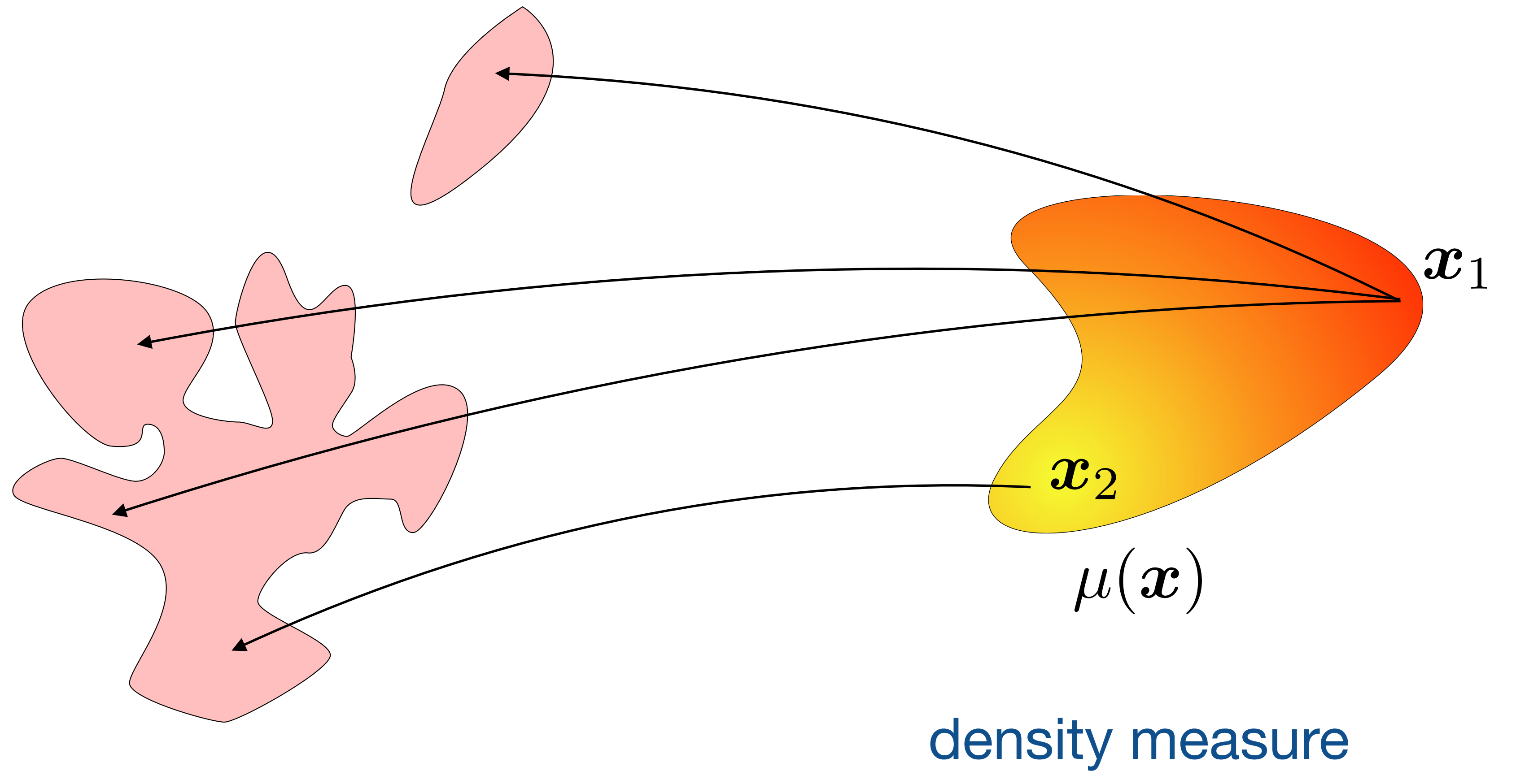
$$
\begin{array}{lll}
\min\limits_{\underline{\boldsymbol{x}},\underline{\boldsymbol{u}},(\Delta t_s)} & \displaystyle\sum_{s=1}^{S}\int_{t_s}^{t_s+\Delta t_s} \ell_s(\boldsymbol{x},\dot{\boldsymbol{x}})\,dt & \text{cost function} \\
\text{s.t.} \quad \forall t & \dot{\boldsymbol{x}} = f(\boldsymbol{x},\boldsymbol{u}) & \text{centroidal dynamics} \\
\forall t & \boldsymbol{u} \in \mathcal{K} & \text{forces in friction cones} \\
 & \boldsymbol{x}(0) = \boldsymbol{x}_0 & \text{initial state is given} \\
 & \boldsymbol{x}(T) \in \mathcal{X}_* & \text{final state must be viable}
\end{array}
$$

The feasibility problem

Working with reduced dynamics is equivalent to a PROJECTION implying a **loss of information**



DENSITY MEASURE **stores** a part of the lost information



Tailored reduced optimal control problem

feasibility measure

$$\min_{\underline{\boldsymbol{x}},\underline{\boldsymbol{u}},(\Delta t_s)} \sum_{s=1}^{S} \int_{t_s}^{t_s+\Delta t_s} \ell_s(\boldsymbol{x},\dot{\boldsymbol{x}}) \cancel{dt} \log\, \mu_s(\boldsymbol{x},\dot{\boldsymbol{x}})\, dt$$

cost function

$$\text{s.t.} \quad \forall t \quad \dot{\boldsymbol{x}} = f(\boldsymbol{x},\boldsymbol{u})$$

centroidal dynamics

$$\forall t \quad \boldsymbol{u} \in \mathcal{K}$$

control in friction cones

$$\boldsymbol{x}(0) = \boldsymbol{x}_0$$

initial state is given

$$\boldsymbol{x}(T) \in \mathcal{X}_*$$

final state must be viable

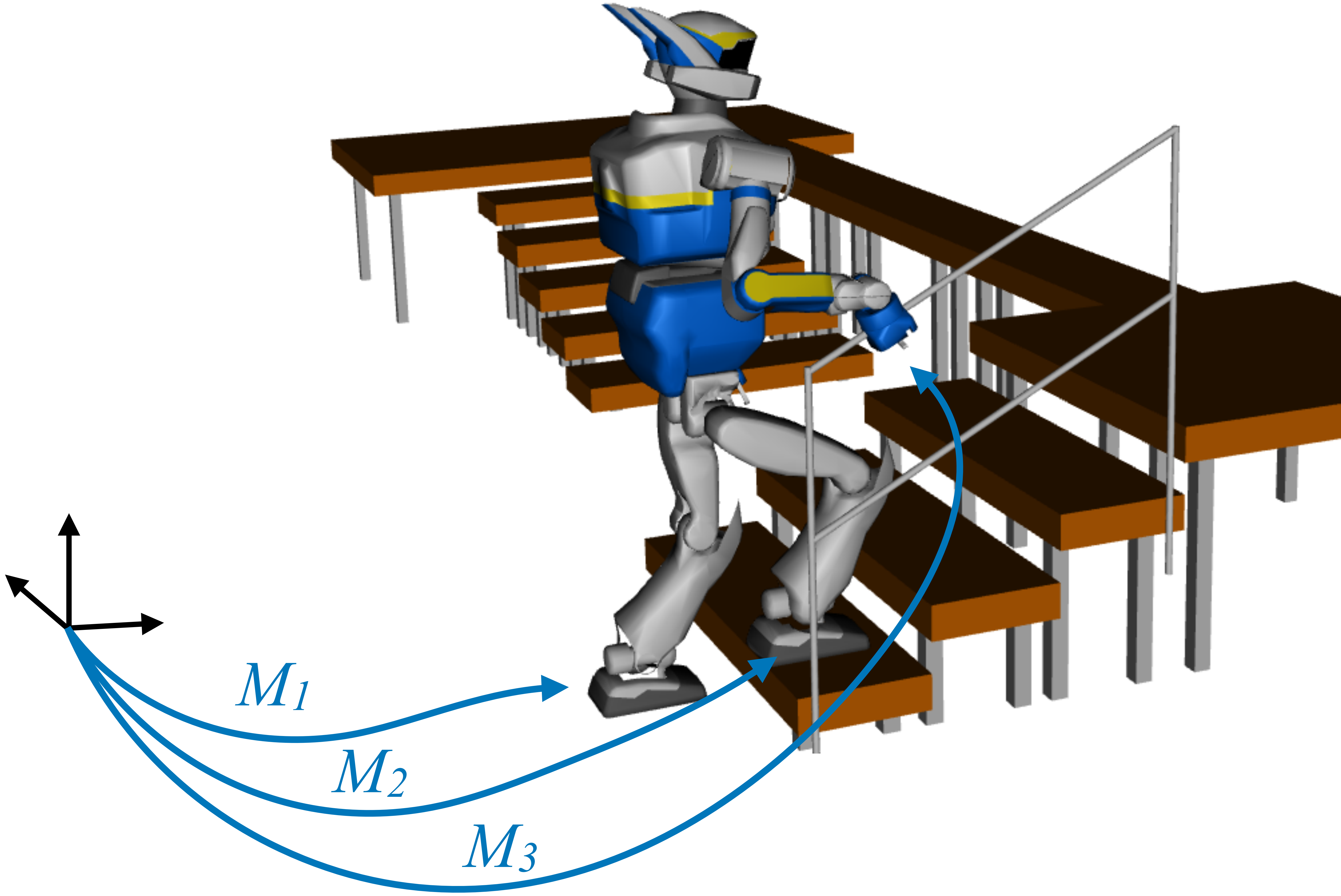
$$\cancel{\forall t \quad \exists\, (\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) \text{ s.t. } \boldsymbol{x},\dot{\boldsymbol{x}} \text{ is feasible}}$$

feasibility constraint

Learning the center of mass feasibility measure

$$p(\boldsymbol{c}|M_1, M_2, ...)$$

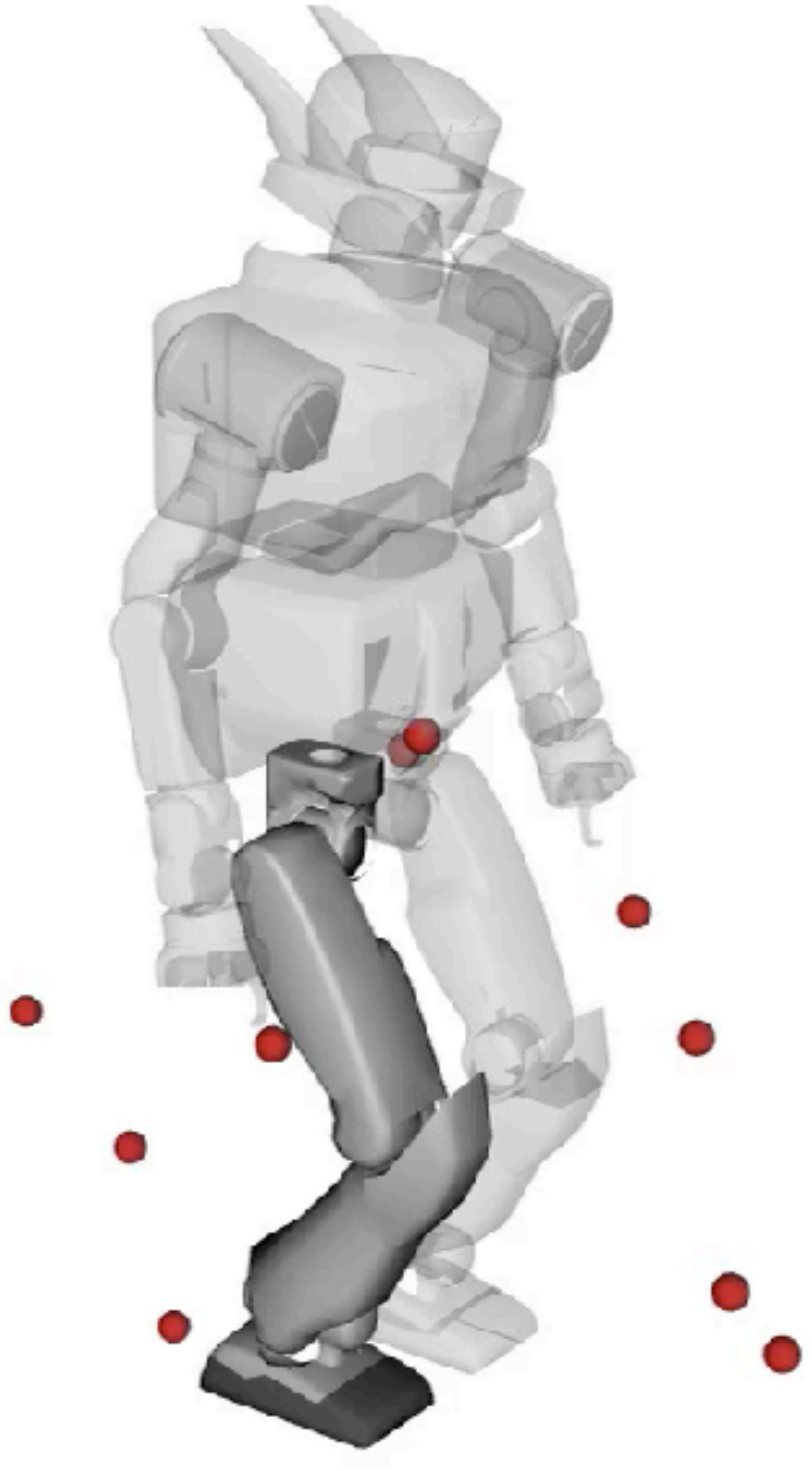
Placement of the *1st* end-effector



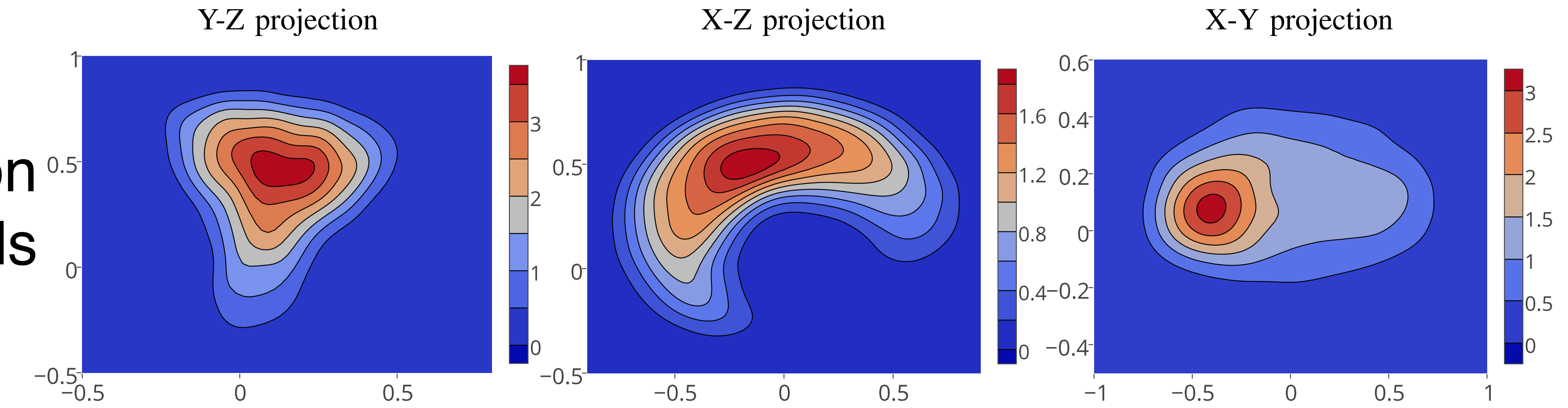
We assume each end-effector is **independent** from each other.

Learning the center of mass feasibility measure in practice

We learn the probability distribution of the CoM to be in the end-effector frame.



Density estimation
20 000 kernels



Final formulation of the problem

$$
\begin{aligned}
\min_{\underline{\boldsymbol{x}},\underline{\boldsymbol{u}},(\Delta t_s)} \quad & \sum_{s=1}^{S} \int_{t_s}^{t_s+\Delta t_s} \ell_s(\boldsymbol{x}, \dot{\boldsymbol{x}}) - \log\ \mu_s(\boldsymbol{x}, \dot{\boldsymbol{x}})\, dt && \text{cost function} \\
\text{s.t.} \quad & \forall t \quad \dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}) && \text{centroidal dynamics} \\
& \forall t \quad \boldsymbol{u} \in \mathcal{K} && \text{control in friction cones} \\
& \boldsymbol{x}(0) = \boldsymbol{x}_0 && \text{initial state is given} \\
& \boldsymbol{x}(T) \in \mathcal{X}_* && \text{final state must be viable}
\end{aligned}
$$

Efficient resolution of the optimal control problem

Solver specifications:

- warm starting of the problem
- exploiting sparsity induced by time
- durations of phases must be optimized
- handle an unstable dynamics

Potential methods:

- collocation
- differential dynamic programming
- single shooting
- multiple-shooting

MUS II COD

[Leineweber et al., CCE, ’03]

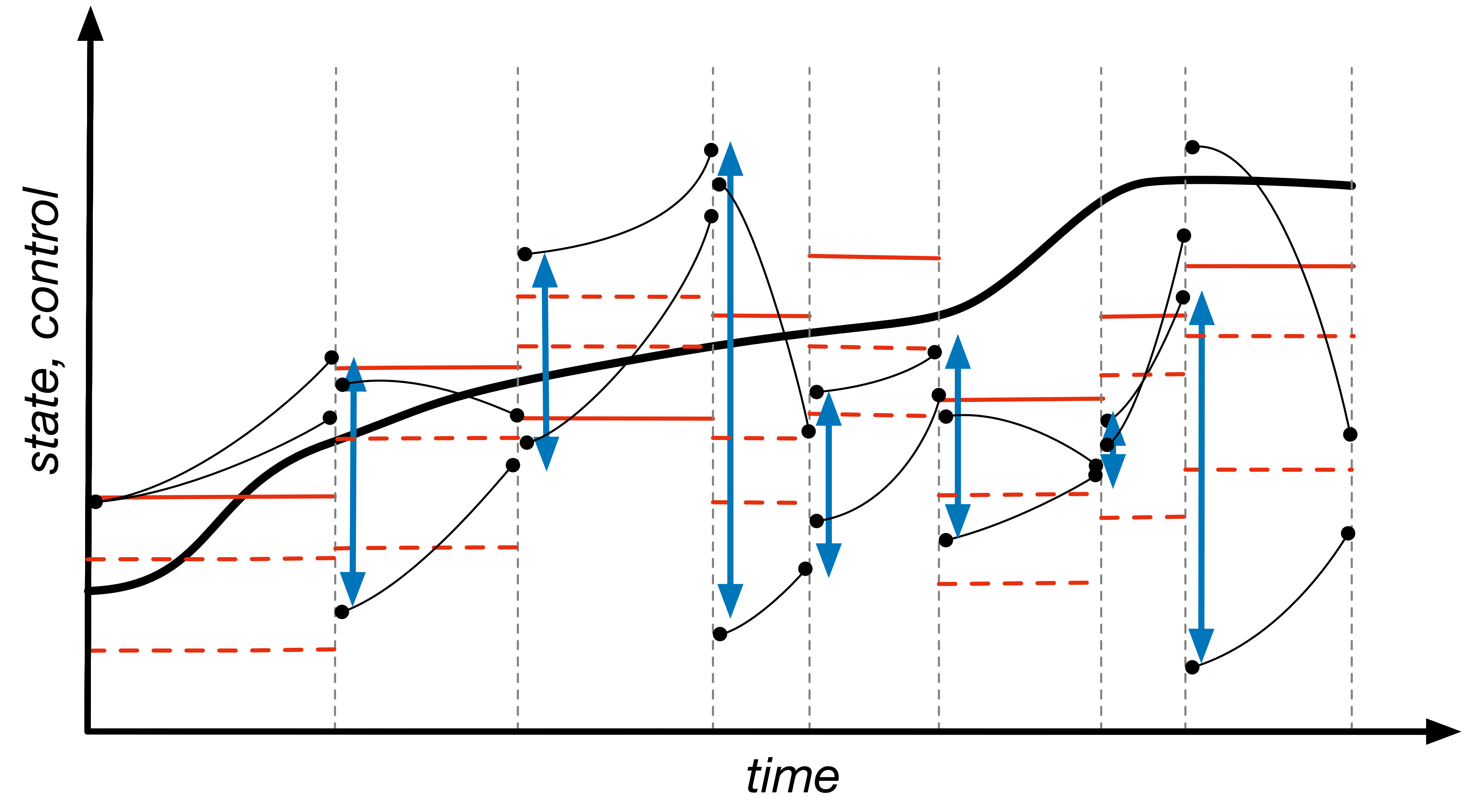
Implementation of a dedicated multiple-shooting solver

Generic optimal control problem
to solve:

$$\begin{aligned} \min_{\underline{\boldsymbol{x}},\underline{\boldsymbol{u}}} \quad & \int_0^T \ell(\boldsymbol{x},\boldsymbol{u})\,dt \\ \text{s.t.} \quad \forall t \quad & \dot{\boldsymbol{x}} = f(\boldsymbol{x},\boldsymbol{u}) \\ \forall t \quad & g(\boldsymbol{x},\boldsymbol{u}) \geq \boldsymbol{0} \\ & \boldsymbol{x}(0) = \boldsymbol{x}_0 \end{aligned}$$

Transforming into

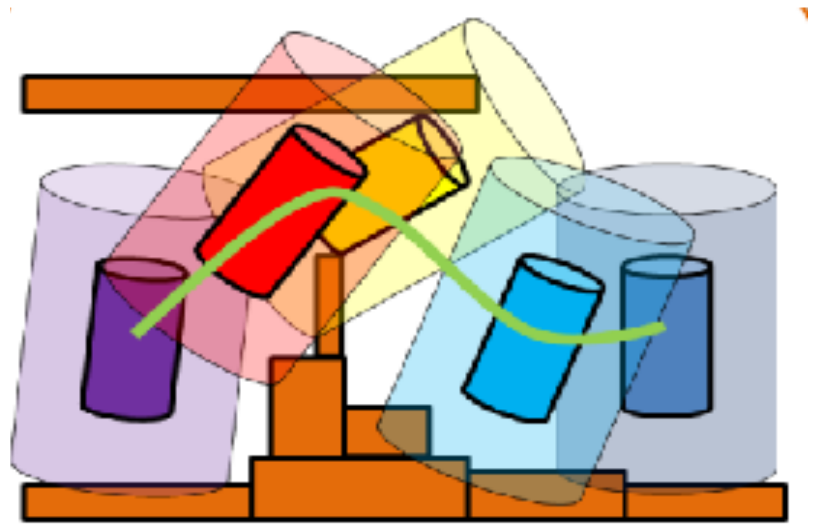
a non-linear problem

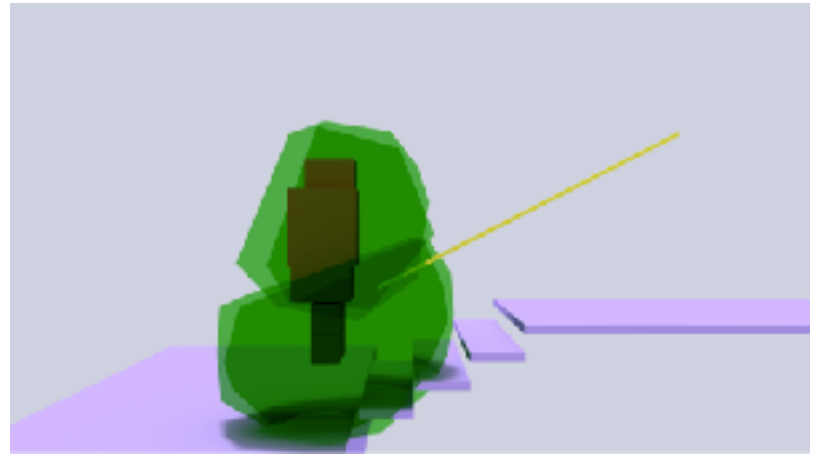


Whole-body motion generation

> 1 Hz

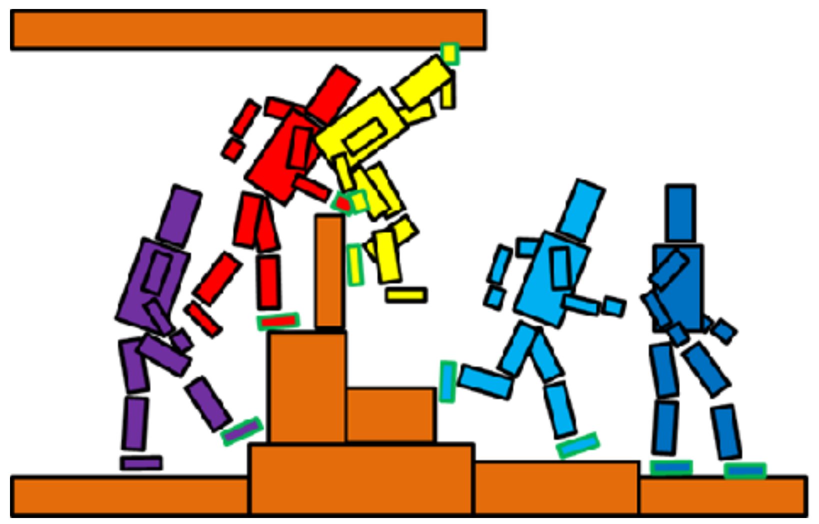
Root path planner

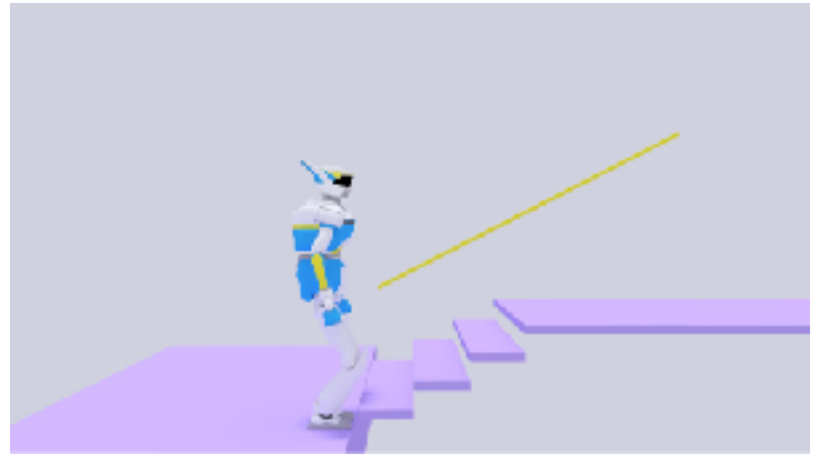




2-5 Hz

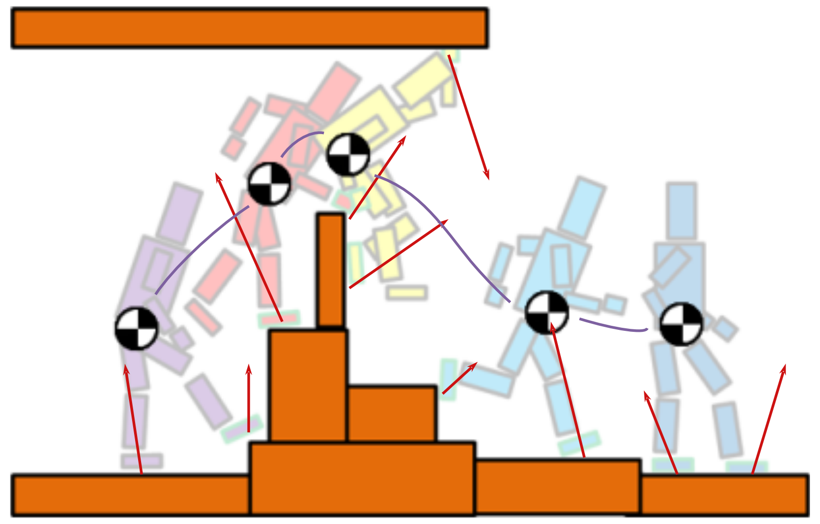
Contact sequence planner

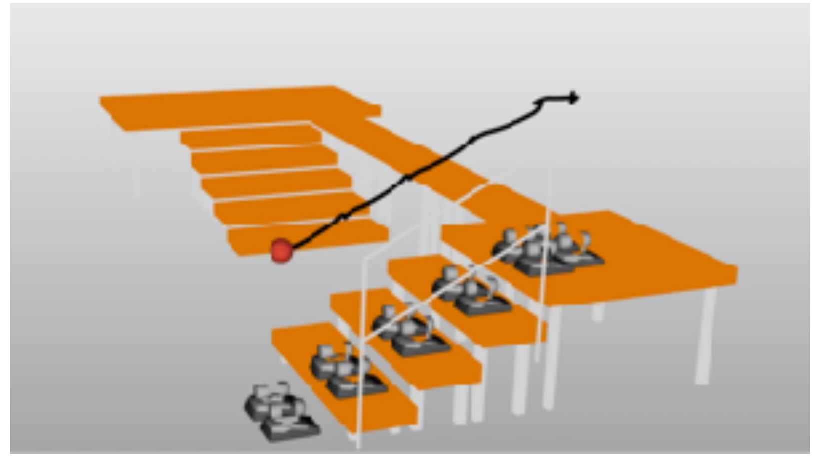




100 Hz

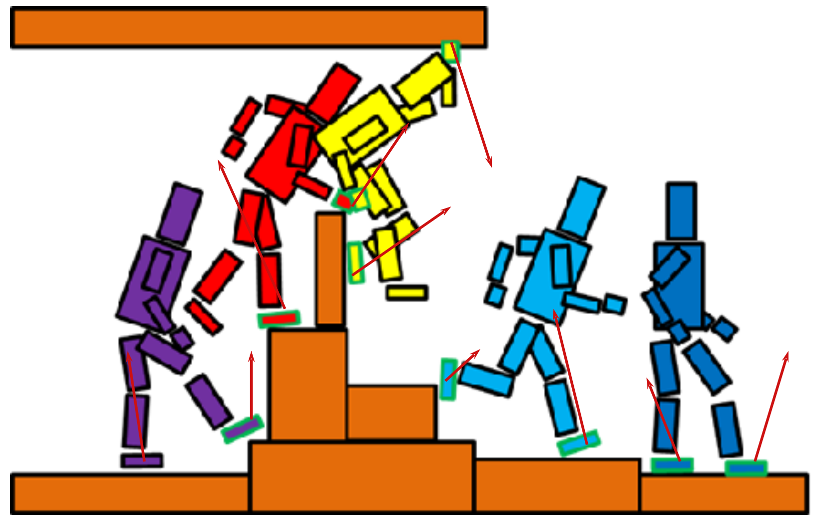
Centroidal motion generation





1-2 kHz

Whole-body motion generation



Whole-body motion generation

Recall that the whole-body dynamics is given by:

$$
\begin{aligned}
M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + g(\boldsymbol{q}) &= S\boldsymbol{\tau} + J(\boldsymbol{q})^\top \boldsymbol{\lambda} \\
J(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{J}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} &= \mathbf{0}
\end{aligned}
$$

We solve a second optimal control problem:

$$
\begin{aligned}
\min_{\underline{\boldsymbol{x}},\underline{\boldsymbol{u}}} \quad & \int_0^T \ell_{\text{tracking}}(\boldsymbol{x},\boldsymbol{u}) + \ell_{\text{reg}}(\boldsymbol{x},\boldsymbol{u})\, dt \\
\text{s.t.} \quad \forall t \quad & \dot{\boldsymbol{x}} = f(\boldsymbol{x},\boldsymbol{u}) \\
& \boldsymbol{x}(0) = \boldsymbol{x}_0
\end{aligned}
$$

Differential Dynamics Programming

Discrete version of the optimal control problem:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + f(\boldsymbol{x}_k, \boldsymbol{u}_k)\Delta t$$

discretized dynamics

$$J_i(\boldsymbol{x}, \boldsymbol{u}_i, \cdots, \boldsymbol{u}_{N-1}) = \sum_{k=i}^{N-1} \ell(\boldsymbol{x}_k, \boldsymbol{u}_k)\Delta t + \ell_f(\boldsymbol{x}_N) \mid \boldsymbol{x}_i = \boldsymbol{x}$$

cost-to-go at time i

$$V_i(\boldsymbol{x}) = \min_{\boldsymbol{u}_i, \dots, \boldsymbol{u}_{N-1}} J_i(\boldsymbol{x}, \boldsymbol{u}_i, \cdots, \boldsymbol{u}_{N-1})$$

value function at time i

Bellman equation:

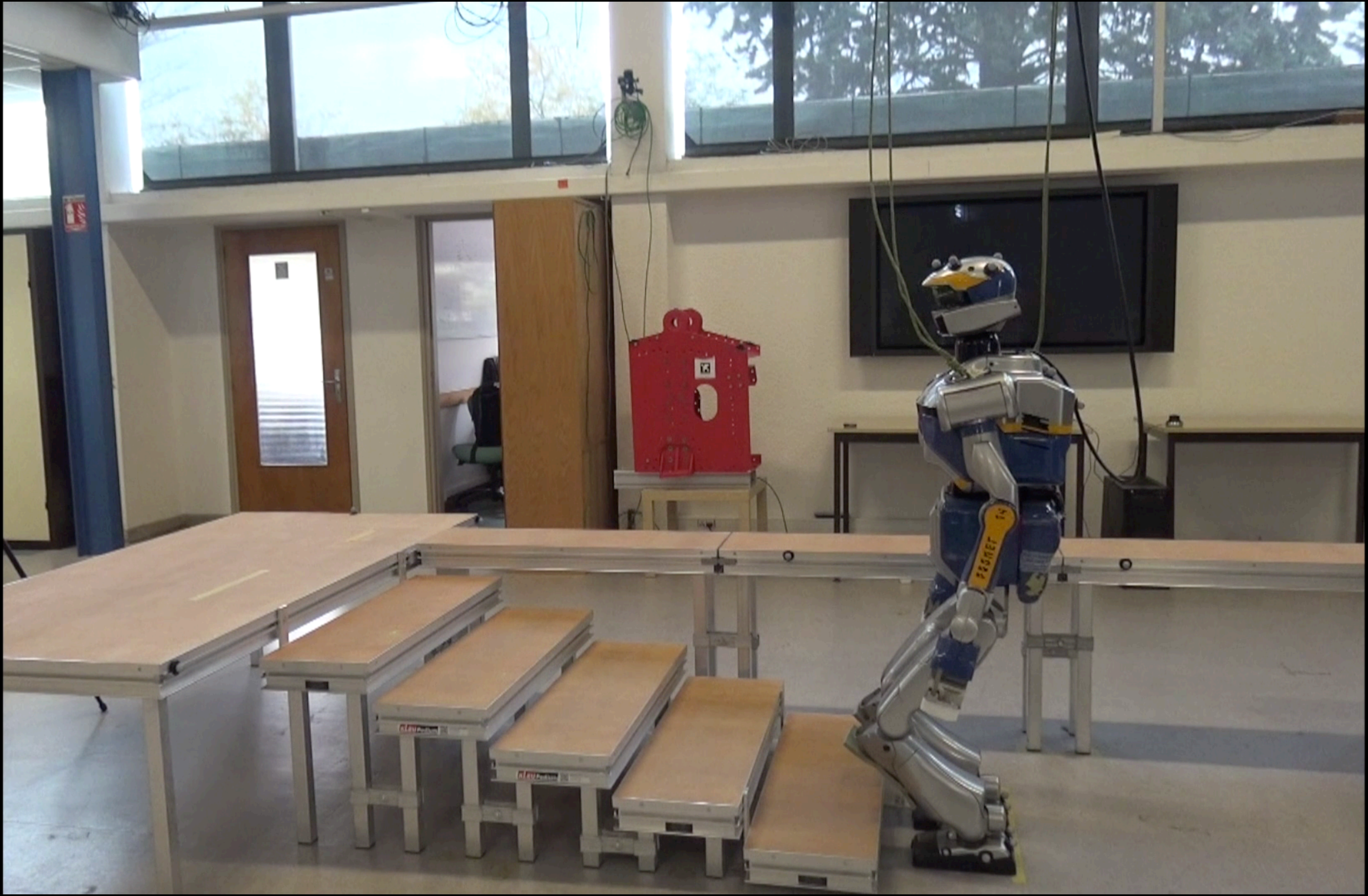
$$V_i(\boldsymbol{x}) = \min_{\boldsymbol{u}_i} \left[\ell(\boldsymbol{x}, \boldsymbol{u}_i) + V_{i+1}(\boldsymbol{x} + f(\boldsymbol{x}_i, \boldsymbol{u}_i)\Delta t) \right]$$

Solving a quadratic version of the Bellman equation:

$$\Delta \boldsymbol{u}_i = \boldsymbol{k}_i + K_i \Delta \boldsymbol{x}_i$$

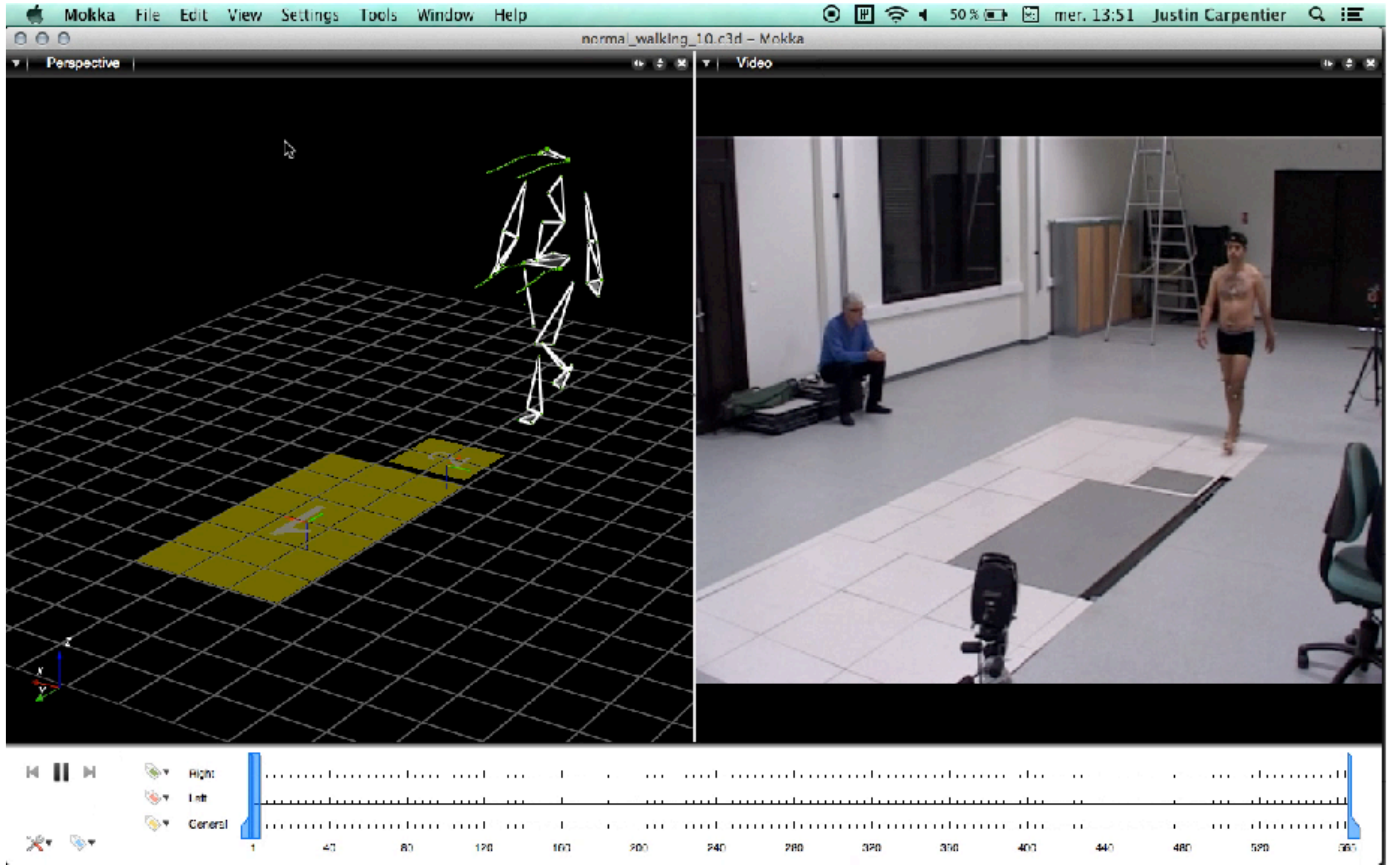
feed-back control

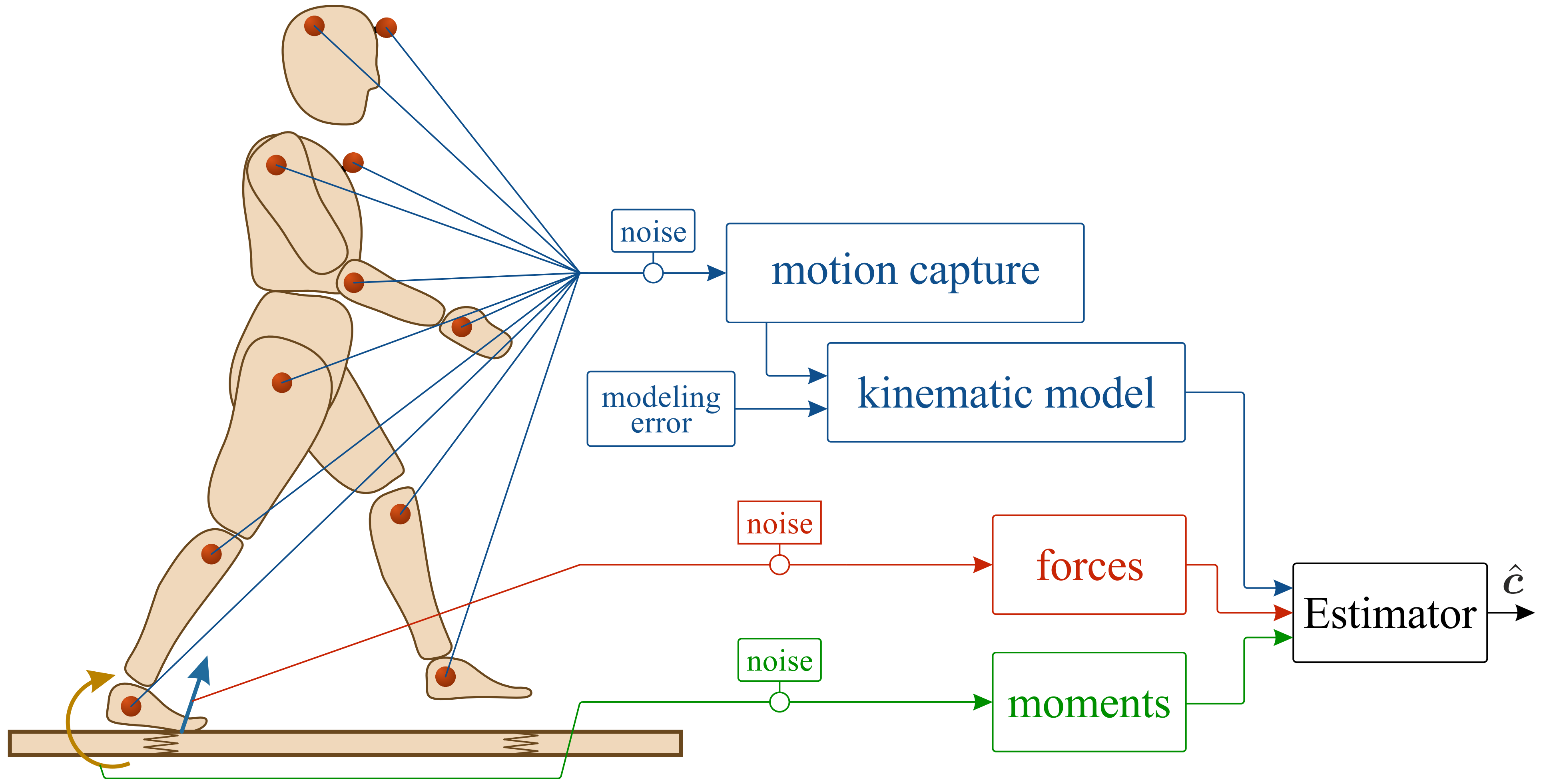
Experimental results



Duality between control and estimation

Duality between estimation and control






Gepetto